AQA

A-level
FURTHER MATHEMATICS
7367/3D

Paper 3 Discrete

Mark scheme

June 2024

Version: 1.0 Final

2 4 6 A 7 3 6 7 / 3 D / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

No student should be disadvantaged on the basis of their gender identity and/or how they refer to the gender identity of others in their exam responses.

A consistent use of 'they/them' as a singular and pronouns beyond 'she/her' or 'he/him' will be credited in exam responses in line with existing mark scheme criteria.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)
ISW	Ignore Subsequent Workings

Examiners should consistently apply the following general marking principles:

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Ticks 3rd box	1.1b	B1	{0, 1, 2, 3} Addition modulo 4
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Ticks 3rd box	1.1b	B1	19, 48
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Ticks 4th box	1.1b	B1	*G* is planar
	Question total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
4(a)	Identifies the four correct row minima or four correct column maxima	3.1a	M1	row minima: $-2, -4, -1, -3$ column maxima: 5, 1, 2, 4 max(row minima) $= -1$ min(column maxima) $= 1$ As max(row minima) $= -1 \neq 1$ $=$ min(col maxima), therefore, a stable solution does not exist.
	States max(row minima) $= -1$ and min(column maxima) $= 1$	1.1b	A1	
	Completes a reasoned argument to show that the max(row minima) and min(column maxima) are not equal and concludes that a stable solution does not exist	3.2a	R1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
4(b)	Deduces **C** for Daniel and **X** for Jackson	2.2a	B1	Play-safe strategy for Daniel $=$ **C** Play-safe strategy for Jackson $=$ **X**
	Subtotal		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(i)	Sets up a model of finding a minimum spanning tree with 9 arcs and at least 5 arcs correct	3.3	M1	*X*–*A*: 100 *A*–*D*: 75 *D*–*G*: 95 *G*–*H*: 75 *D*–*E*: 100 *B*–*E*: 65 *E*–*I*: 95 *F*–*I*: 45 *C*–*E*: 125
	Finds the correct minimum spanning tree	3.4	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)(ii)	Obtains 775 hours FT their minimum spanning tree	3.2a	B1F	$100 + 75 + 95 + 75 + 100 + 65 + 95 + 45 + 125$ $= 775$ hours
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
5(b)	Recognises a valid limitation of the model in the context of the question related to the connectedness of the network	3.5b	E1	If an electrical connection fails, it may result in more than one car park being unable to charge electric cars.
	Subtotal		**1**	

	Question total		**4**	

Q	Marking instructions	AO	Marks	Typical solution
6	Sets up a model by identifying the problem as a route inspection problem and noting that *A*, *G*, *J* and O are odd-degree nodes (PI)	3.3	M1	Odd degree nodes: *A*, *G*, *J*, *O* Shortest Distances *A*–*G*: 2.7 *J*–*O*: 2.0 *A*–*J*: 2.4 *G*–*O*: 2.5 *A*–*O*: 3.9 *J*–*G*: 2.4 Pairings (*A*–*G*)(*J*–*O*) = 4.7* (*A*–*J*)(*G*–*O*) = 4.9 (*A*–*O*)(*J*–*G*) = 6.3 Minimum total distance the van must travel is $31.4 + 4.7 = 36.1$ miles The minimum amount of fuel the van requires is $\frac{36.1}{7.8} = 4.63$ litres Therefore, the van will require more than 4.5 litres of fuel and so does not have enough fuel to make all of its deliveries and arrive back at the junction it started from.
	Uses the model to find at least one correct total for a pair of shortest distances	3.4	M1	
	Finds all three correct totals for the pairs of shortest distances	1.1b	A1	
	Determines their correct minimum total distance that the van needs cover during the journey or Determines that the maximum distance the van can travel with 4.5 litres of fuel is 35.1 miles	1.1b	B1F	
	Determines the minimum amount of fuel that would be required or Makes a comparison of their route with 35.1 miles	1.1b	B1F	
	Uses the model to correctly conclude that the van does not have enough fuel to make all of its deliveries and arrive back at the junction it started from	3.5a	E1F	
	Question total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Sets up test for associativity by considering combinations of the form $(x-y)-z$ and $x-(y-z)$, where $x, y, z \in \mathbb{Z}$, and simplifies one combination	1.1a	M1	$(3-2)-1=0$ $3-(2-1)=2$ As $(3-2)-1 \neq 3-(2-1)$, the set of integers does not possess the associativity property. The associativity property is required for all groups, therefore the set of integers does not form a group under subtraction.
	Completes a reasoned argument by showing a full counterexample to associativity (either algebraic or numerical) and concludes that the set of integers does not form a group under subtraction	2.1	R1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(b)(i)	Obtains at least one fully correct row or at least one fully correct column of the Cayley table	1.1a	M1	See below
	Obtains at least three fully correct rows or at least three fully correct columns of the Cayley table	1.1a	M1	
	Completes the Cayley table correctly	1.1b	A1	

$\times_{19}$	1	7	8	11	12	18
1	1	7	8	11	12	18
7	7	11	18	1	8	12
8	8	18	7	12	1	11
11	11	1	12	7	18	8
12	12	8	1	18	11	7
18	18	12	11	8	7	1

	Subtotal		**3**	

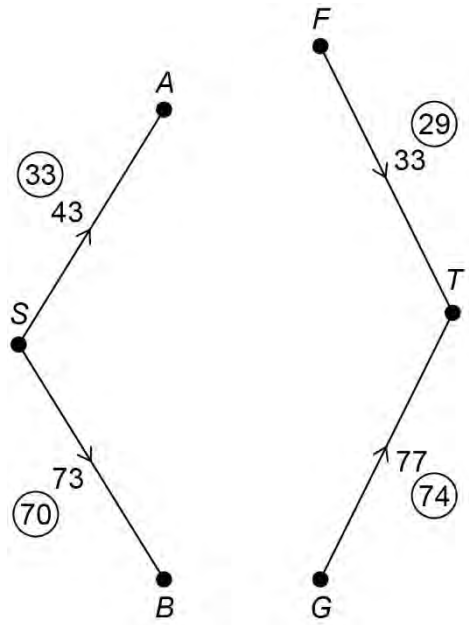
Q	Marking instructions	AO	Marks	Typical solution
7(b)(ii)	States 7	2.2a	B1	7
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(i)	States at least two possible correct orders for the proper subgroups of *G*	1.1a	M1	By Lagrange's theorem, the order of a subgroup must divide the order of the group. Hence, the possible orders for the proper subgroups of *G* are 1, 2 & 3 as these are divisors of 6, the order of *G*
	States the orders of proper subgroups of *G* as 1, 2 & 3 only (not 6) and explains that these are divisors of 6 and names or explains Lagrange's theorem	2.4	A1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(ii)	Finds at least one correct proper subgroup of *G* Condone poor notation or answer given in different form	1.1a	M1	$\left(\langle 1\rangle, \times_{19}\right)$ $\left(\langle 7\rangle, \times_{19}\right)$ $\left(\langle 18\rangle, \times_{19}\right)$
	Finds at least two correct proper subgroups of *G* Condone poor notation or answer given in different form	1.1b	A1	
	Finds all three distinct proper subgroups of *G* and no others, giving all answers in the correct form Note $\left(\langle 7\rangle, \times_{19}\right)$ and $\left(\langle 11\rangle, \times_{19}\right)$ are not distinct	2.5	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
7(c)(iii)	States a correct full name for *H*	1.2	B1	Cyclic group of order 6
	Subtotal		**1**	

	Question total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	Connects supersource *S* to nodes *A* and *B* with directed arcs and connects supersink *T* to nodes *F* and *G* with directed arcs	1.1b	B1	
	Includes a correct upper capacity on each of the four correct arcs	1.1b	B1	
	Subtotal		**2**	

Q	Marking instructions	AO	Marks	Typical solution
8(b)	Finds at least one correct augmenting path and the extra flow (may be seen on diagram). Condone *S* & *T* not included in augmenting path	3.1a	M1	<table><tr><th>Augmenting Path</th><th>Extra Flow</th></tr><tr><td>*SADFT*</td><td>4</td></tr><tr><td>*SBGT*</td><td>2</td></tr><tr><td>*SBCEGT*</td><td>1</td></tr></table>
	Finds a second correct augmenting path and the extra flow. Condone *S* & *T* not included in augmenting path	1.1b	A1	Maximum flow = 110 litres per second
	Finds a third correct augmenting path and the extra flow, and no incorrect paths. Total of all extra flows must be 7 Condone *S* & *T* not included in augmenting path	1.1b	A1	
	Obtains 110	2.2a	B1	
	Subtotal		**4**	

Q	Marking instructions	AO	Marks	Typical solution
8(c)	Explains that *EG* is saturated/at maximum flow before the leak	2.4	E1	*EG* is saturated, and none of the flow through *EG* can be re-routed as each of *BG*, *DG* and *DF* are also saturated. Hence, the maximum flow through the network decreases by 31 litres per second and so the engineer's claim is correct.
	Explains that the flow of 31 litres per second through *EG* cannot be re-routed as *BG*, *DG* & *DF* are also all saturated or that *EG* is part of the minimum cut of the network and then concludes that the engineer's claim is correct	2.3	R1	
	Subtotal		**2**	

	Question total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Explains that $\mathbf{J_4}$ is dominated by $\mathbf{J_1}$ OE	2.4	B1	As $1 \leq 2$, $6 \leq 7$ and $4 \leq 6$, strategy $\mathbf{J_1}$ dominates strategy $\mathbf{J_4}$
	Subtotal		**1**	

Q	Marking instructions	AO	Marks	Typical solution
9(b)(i)	Introduces four slack variables in the column headings and finds at least two correct rows	3.1a	M1	See below
	Finds all rows correctly	1.1b	A1	

P	v	P_1	P_2	P_3	r	s	t	u	**value**
1	–1	0	0	0	0	0	0	0	0
0	1	–2	–5	–4	1	0	0	0	0
0	1	–7	–5	–3	0	1	0	0	0
0	1	–6	–1	–8	0	0	1	0	0
0	0	1	1	1	0	0	0	1	1

	Subtotal		**2**	

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td rowspan="2">9(b)(ii)</td><td>Uses the simplex algorithm to modify at least one non-pivot row correctly</td><td>3.1a</td><td>M1</td><td rowspan="2">See below</td></tr>
<tr><td>Uses the simplex algorithm to find all rows correctly</td><td>1.1b</td><td>A1</td></tr>
<tr><td></td><td colspan="4">
<table>
<tr><th>P</th><th>v</th><th>P_1</th><th>P_2</th><th>P_3</th><th>r</th><th>s</th><th>t</th><th>u</th><th>value</th></tr>
<tr><td>1</td><td>0</td><td>–2</td><td>–5</td><td>–4</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>1</td><td>–2</td><td>–5</td><td>–4</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>–5</td><td>0</td><td>1</td><td>–1</td><td>1</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>–4</td><td>4</td><td>–4</td><td>–1</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>0</td><td>0</td><td>1</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>1</td><td>1</td></tr>
</table>
</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>2</td><td></td></tr>
</table>

<table>
<tr><th>Q</th><th>Marking instructions</th><th>AO</th><th>Marks</th><th>Typical solution</th></tr>
<tr><td>9(c)</td><td>Obtains $\frac{5}{12}$</td><td>2.2a</td><td>B1</td><td>$\frac{5}{12}$</td></tr>
<tr><td></td><td>Subtotal</td><td></td><td>1</td><td></td></tr>
</table>

<table>
<tr><td></td><td>Question total</td><td></td><td>6</td><td></td></tr>
</table>

Q	Marking instructions	AO	Marks	Typical solution
10(a)	States *BDGJK* and no others	1.1b	B1	*BDGJK*
	Subtotal		**1**	

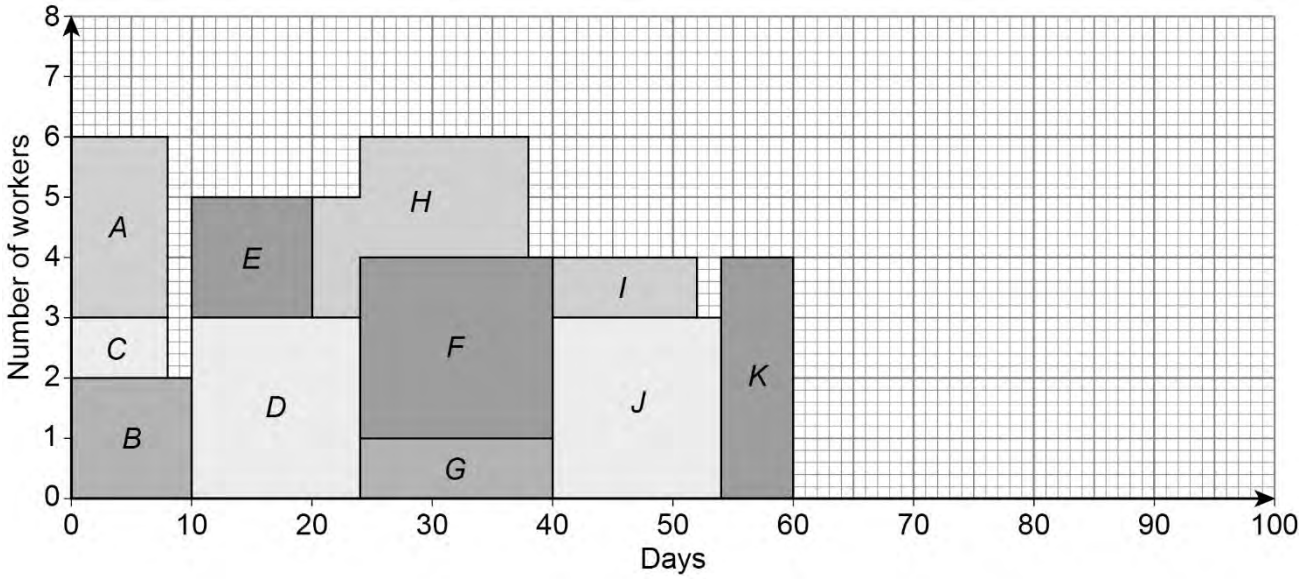
Q	Marking instructions	AO	Marks	Typical solution
10(b)	Draws a resource histogram with at least 10 labelled activities	3.1b	M1	See next page
	Draws a resource histogram with activities *A*, *B*, *C*, *D* & *E* drawn correctly	1.1b	A1	
	Draws a fully correct resource histogram	1.1b	A1	
	Subtotal		**3**	

Q	Marking instructions	AO	Marks	Typical solution
10(c)	Shows that activities *A*, *B* and *C* cannot be carried out simultaneously on histogram	3.1b	M1	See next page 96 days
	Draws a fully correct, labelled and levelled resource histogram	1.1b	A1	
	Obtains 96 days Condone missing units	3.2a	A1	
	Subtotal		**3**	

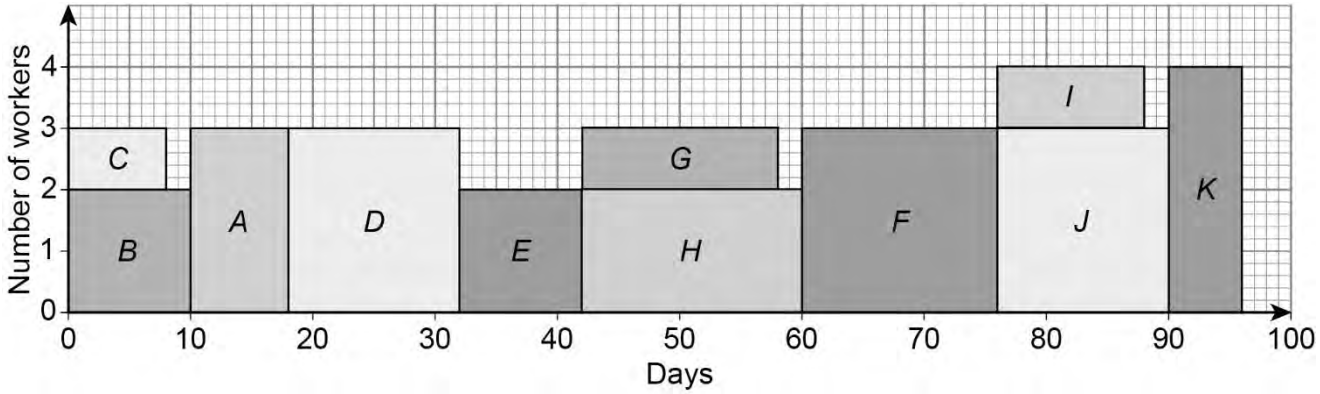
	Question total		**7**	

	Question Paper total		**50**	

10(b)


Number of workers
Days
A
B
C
D
E
F
G
H
I
J
K

10(c)


Number of workers
Days
A
B
C
D
E
F
G
H
I
J
K